Mid-Semestral Examination I Semester 2002-2003
B. Math. Hons. I Year
Analysis I

Date: 07-10-2002 **Marks: 50** **Time: 3 Hours**

Answer all the questions.

1. Let X be a countable set. Show that X is infinite iff there is a proper subset $A \subset X$ and a one-one and onto map $f : A \to X$. [10]

2. Let $\Omega = \{a + ib : a, b \in Q\}$. Show that Ω is a countable infinite set. [5]

3. Show that any nested sequence of closed interval has non-empty intersection. [5]

4. Let $\{a_n\}$ be a sequence of positive numbers. If $\lim \frac{a_{n+1}}{a_n} = 1$ show that $\lim \sqrt[n]{a_n} = 1$ Give complete details. [5]

5. With detailed proofs discuss the convergence or divergence of the series $\sum \frac{1}{1+z^n}$. [10]

6. Suppose $a_n > 0, \sum a_n$ diverges. Show that there exists a decreasing sequence $\alpha_n \to 0$ such that $\sum \alpha_n a_n$ diverges. Give an example to show that '$a_n > 0$' cannot be dropped. [10]

7. Let $a_n, b_n > 0, \sum a_n = A, \sum b_n = B$. Let C_n be the nth term of the Cauchy product. Show that $\sum C_n = AB$. [5]